Gyroscopes
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1 Abstract

In this experiment we measured the moment of inertia of an aluminum disk using two different methods, both of which convert the system into a working pendulum, the value for the moment of inertia calculated was $I = 0.0113 \pm 0.002 kgm^2$ more is displayed in the results section. Furthermore using the methods stated above, the moment of inertia of a bicycle wheel was calculated to be $I = 0.14377 \pm 0.016 kgm^2$, we then used the moment of inertia and the observed period of oscillation to calculate the frequency of precession theoretically the results are displayed in the results section, on the other hand we plotted the relationship of the precession frequency and the angular frequency against the distance d (distance from the balance distance of the gyroscope), and obtained a gradient which was accurate within 5% to the true value giving us confidence in the methodology used and confidence that the theoretical value accurately described the dynamics of the gyroscopes. Finally for the nutation of the gyroscope we discover the the relationship between a nutating and a non nutating gyroscope's period of oscillation ia a direct linear relationship, where the non nutating angular frequency is always greater that the nutating angular frequency, the percentage difference between the two was observed to always lay between -9& and -7%. More results can be found in section 4.

2 Introduction

Gyroscopes are physical bodies that rotate, usually they consist of a thin disk and a pole that supports the disk at it's center, the disk is the only component that rotates.The spinning is induced by applying a force to the disk tangential to it's edges, this force will cause the disk to spin.
We will first attempt to gain information about the system so we can investigate gyroscopes further.We will attempt to measure the Moment of inertia of a solid aluminum disk, this will allow us to gain an understanding of the procedures used to find the moment of inertia of objects.Furthermore we will investigate the Precession and nutation of the Gyroscope which are two properties of the rotating system. Gyroscopes were invented by different civilizations including, classical Greece, Rome, and China but were never used as instruments. The known time in which gyroscopes were known to be used as instruments was in 1743 when Jhon Serson used a gyroscope as a level to indicated the horizontal ground [2]. Today Gyroscopes have many applications e.g Accelerometers,Compass/Gyrocompass and as heading indicators useful in air craft steering. The procedure we will use to investigate gyroscopes can be found in section 3.

3 Procedure

3.1 Equipment [1]

- Gyroscope
- digital counter
- 2x multicore cables
- stand abse with gyroscope support pole
- 2x 100g weights
- Vernier callipers
- dynamo-meter with scale up to 100N
- torsion axle with V-shape stand base
- stopwatch
- circular Aluminum disk with perforations

3.2 Objectives

1) Measure the moment of Inertia of an aluminum disk.
2) Investigate the Precession of a gyroscope.
3) Investigate the nutation of a gyroscope.

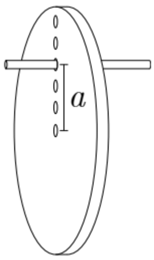
3.3 Method

For objective 1:
We start by investigating an aluminum disk with a radius of 18cm, we attache a weight of $195.77 \pm 0.02g$ to its edge and measure the distance from the center of the disk to the center of the weights this distance was observed to be $16.3 \pm 0.05cm$, further more we allowed the disc which may now be considered to be a pendulum to swing so we can measure the period of oscillation and use equation 1 to calculate the moment of inertia of the system.

$$I = \frac{M_{wt} R_{wt} g \delta t^2}{4\pi^2} - M_{wt} R_{wt}^2 \tag{1}$$

where M = mass of weight, R = radius of weight, g = gravitational acceleration and t= period of oscillation.
Further more we will investigate the moment of inertia of the same Aluminum disk using a different method 'Stein's law' in which there wont be any wights attached to the disc and instead of the axis of rotation being through the center of the disc it will be at varying distances from the center as shown in figure 1 where a = (4,8,12,16)cm.

Figure 1: Showing how the position of the axis of rotation may change [1]



The mass of the disc was measured to be $729.55 \pm 0.02g$, the period of oscillation is again measured by measuring the time taken for the pendulum to oscillate 6-7 times then working out the average period for one oscillation, for precaution we don't start measuring the period until the first oscillation has passed, this is to reduce human error due to reaction time as well as input force from the hand giving the disc a push.We the proceed to find the moment of inertia of the disc at every distance using equation 1 again where instead of R being the radius it's now the distance from the axis of rotation to the center of the disc, the results obtained are compared with the theoretical value calculated by equation 2 [1].

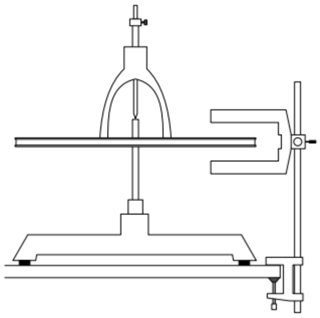
$$I_{cyl} = \frac{M_{cyl} R_{cyl}^2}{2} \tag{2}$$

For objective 2:
To be able to investigate the precession of the gyroscope, we will use a bicycle wheel with 18 spokes equally spaced around the wheel, then we will proceed to measure the moment of inertia of the wheel by attaching the $195.77 \pm 0.02g$ weight and by allowing the system to swing back and forth like a pendulum we can measure the period and hence by using equation 2 where M = 195.77g, R= radius of wheel and δt is the period of oscillation of the system we can calculate the moment of inertia. Once we obtain the moment of inertia of the system we can use equation 3 to calculate a theoretical value for the frequency of precession,

$$w_{precession} = \frac{mgd}{Iw} \tag{3}$$

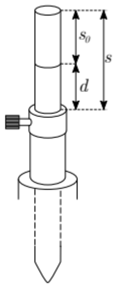
Where m = M = 195.77g, g = gravitational acceleration, d = distance, I = moment of inertial of the wheel, w = angular frequency. The angular frequency of the actual disc rotation is measured by measuring the period of oscillation of the system when the center of mass is at the same point on which the gyroscope is balanced (figure 2);

Figure 2: showing how the gyroscope setup to measure the angular frequency [1]



We can obtain the period of oscillation of the rotating system which we can inverse as such $w = \frac{1}{\delta t}$ to obtain the angular frequency. Then finally we substitute all the relevant variables into equation 3 to obtain the frequency of precession. We then repeat this calculation at different values of w 3 times and repeat this process at different distances d 3, the distance s is measure using the Vernier callipers accurate to 0.001mm, it's subtracted from s_o which is constant at all time to obtain the distance d.

Figure 3: measuring of the distance s_0 = balance distance 'constant', s = total distance, d = effective distance [1]



The results include a plot of $w_{precession}$ vs w and a plot of $w_{precession}$w vs d the gradient of which is equal to $\frac{mg}{I}$.

For objective 3:
To investigate the nutation, we set up the gyroscope as shown in figure 2 making sure the gyroscope is balanced i.e the distance d = s_o in figure 3, we then apply a force rotating the wheel, therefore the gyroscope will be spinning and it will be balanced, then we apply an additional angular momentum by hitting/tapping the gyroscope on the edge in a direction parallel to the axis of rotation i.e from top to bottom in figure 2. This additional angular momentum will cause the axis of rotation to oscillate, hence two modes of oscillations are now occurring nutation and the initial normal rotation of the gyroscope. We measured the angular frequency before and after applying the additional angular momentum, thus obtaining two sets of results (w and $w_{nutated}$. These results are shown in section 4 and discussed in section 5

4 Results

4.1 Objective 1

Average period of oscillation:
$\delta t = 1.442 \pm 0.025s$, where the error is the average human reaction time.

Using equation 1 we calculated the value of the moment of inertia of the the aluminum disk: $I = 0.0113 \pm 0.002kgm^2$, where the error in percentage is $\pm 17.69\%$

We proceed to measure the moment of inertia using "Steiner's law" at different distances, results are displayed in table 1:

Table 1: Results for the moment of inertia using Steiner's law

Distance (cm)	Average period t (s)	Moment of Inertia (kgm^2)	% uncertainty in the moment of inertia	Period uncertainty ($\pm$s)
4	1.312	1.13E-02	4.35	2.50E-02
8	1.045	1.12E-02	6.79	
12	1.025	1.24E-02	9.03	
16	1.011	1.10E-02	13.37	

4.2 Objective 2

We measured the average period of oscillation of the bicycle wheel when a 200g weight was attached to it, using this period the moment of inertia was then calculated.

Table 2: Results for the moment of inertia of the bicycle wheel

Distance (cm)	Average period t (s)	Moment of Inertia (kgm2̂)	% uncertainty in the moment of inertia	Period uncertainty ($\pm$s)	Uncertainty in the moment of inertia ($\pm$ kgm^2)
26.00	3.54	0.14377	11.13	0.03	0.016

Furthermore, using the light gate timer we again measured the period of the precessing system and hence the angular frequency of the system, using equation 3 we calculated the frequency of precession. Plots 4 and 5 show the relationship between Wp and w and $Wp \times w$ and the distance d respectively.
We calculated the gradient ($\frac{W_p W}{d}$) from the plot obtained in figure 5, we obtained a value of $222.2hz^2m^-1$ the theoretically calculated value was 214.9 Hz^2m^-1, therefore the value obtained experimentally lays within 5% of the real value which is acceptable.

Figure 4: Showing the relationship between frequency of precession and angular frequency of the wheel

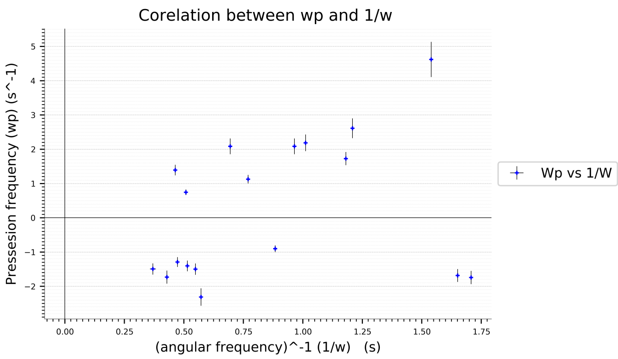
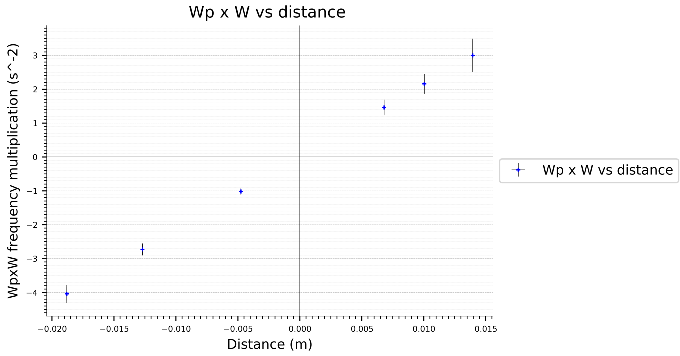


Figure 5



4.3 Objective 3

We calculated the angular frequency before and after we applied an additional angular momentum the results are displayed in table 3.

Table 3: Results for the precessing gyroscope

w (Hz)	w_p (Hz)	% difference (%)
1.73	1.59	-8.35
1.182	1.085	-8.94
2.129	1.984	-7.32

5 Discussion

For objective 1, we obtained a value for the moment of inertia $I = 0.0113kgm^2$ when we attached the 200g mass to the aluminum disk. Furthermore we obtained the values in table 1 for the moment of inertia using a completely different method 'steiner's law 3'. The values for the moment of inertia obtained using the two different methods are very similar for the distance of 4cm almost the exact same value for the moment of inertia (% difference $<$ 1%) was obtained compared to the first result we obtained, and comparing the rest of the calculated values for the moment of inertia in table 1 we can observe that the values are diverging away from the real value (column 4 L->R) suggesting that steiner's law/method has some limitations, where the uncertainty in the value of the calculated moment of inertia increases as the radius increases, this is further discussed in the section 6.

For objective 2, We managed to calculate the angular frequency of the system and used that value to calculate the frequency of precession (method discussed in section 3). Furthermore using these values we were able to plot the relationship between Wp vs w as well as Wp x w vs distance these relationships are displayed in figures 4 and 5 respectively, we observe that there is a direct linear positive correlation between Wp and w meaning that as the gyroscope spins faster the faster it will precess.On the other hand as the distance d increases figure 3 the relationship Wp x w will have a greater value, we also observed that when the distance d is positive i.e d$>$ s_o the gyroscope precessed clock wise, however when the d $< s_s o$ it will precess anti-clock wise, this is due to the position of the center of mass relative the point where the gyroscope is balanced on i.e above or below the point b in figure 2, if it's above the gyroscope will precess clockwise and if it's below precession occur in the anticlockwise direction.

For objective 3, the results in table ref 3 we obtain two values for the angular frequency, with and without nutation, we observe that as the angular frequency increases the nutated angular frequency also increases which is expected, furthermore we observe that, consistently, that the nutated angular frequency is around 8.2% (the average of column 3 L -> R) lower than the non-nutated this is a consistent relationship our range of results.

6 Error analysis

This experiment involved a lot of measurements relient on the human reaction time e.g using the stop watch to measure the period of oscillation of a pendulum, this is a significant source of error and we attempted to reduce it as much as possible e.g by allowing the pendulum to oscillate at least once to allow time for the observer to get ready and expect when to start measuring, we also tried avoiding the use of a stop watch all together by using the Lazer light gate detector which has a much greater precision compared to human. Furthermore other sources of uncertainty include the unaccounted force of friction which is a systematic error, due to the complexity of the system the the force of friction had to be left out as it would have been incredibly difficulty to calculate it in the span of time available.

Other sources of error include anomalies, this is dealt with by simply repeating the measurements a suitable amount of times to ensure certainty that the values obtained are valid, the averages were calculated to obtain values closer to the real value increasing our confidence in the results obtained. When using the Digital counter computer we ensured that the initial values of every measurement sets were discarded, this was done because the actual measuring doesn't start from the correct location i.e the period of the first oscillation obtained maybe half of the true value.We constantly measured the % difference between the theoretical and real values so we obtain confidence in the results, no measurement fell below 20% difference, which given the nature of the experiment and the instability of the system e.g when measuring the period of the pendulum using steiner's law at 16cm, the system proved to be very unstable thus obtaining an uncertainty below 20% in such conditions was considered to be acceptable.

7 Conclusion

To conclude, gyroscopes prove to be systems extremely relient on fundamental physical principles such including torque , moment of inertia, angular momentum and etc, result in a system that displays phenomenon such as precession and nutation and excellent stabilization when calibrated correctly. They are used for many purposes educational or industrial, to teach and stabilize and air craft, making them unique.

8 Python Code

no code for showing off this time...sorry Tim.

References

[1] UKC school of physical sciences. "Gyroscopes Experiment 4". In: (2018). URL: https://moodle.kent.ac.uk/2018/pluginfile.php/156325/mod_folder/content/0/Gyroscopes_Sept%5C%202018.pdf?forcedownload=1.

[2] wiki. "Gyroscope". In: (). URL: https://en.wikipedia.org/wiki/Gyroscope.