Biot Savart law

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4 March 2019

1 Abstract

In this experiment we look at how the Biot savart law predicts the magnetic field strength in and around different current carrying objects, and different orientations,distances,currents and lengths of objects, we discover that as we keep all other variables constant and changing one variable the Biot savart law continues to predict to a fairly accurate degree how the magnetic field strength changes with that variable, with most errors calculated for out experimental results being within 6% of the calculated theoretical value, we can say that our experimental methods are indeed reliable and with certainty we can also state that the Biot savart law accurately describe the reality of how magnetic fields change relative to a certain variable for many different geometries of current carrying objects

2 Introduction

Magnetic fields have been an observed phenomenon since the ancient history of humanity, whats more intriguing is that man-kind has been using magnetic fields to our advantage since (maybe earlier) 625 BC where 'magnetite' has been used to remove arrows from patients in ancient India [2], furthermore we now became advance beings and we use better weapons including a magnetic projectile launcher as well as attempting to make a sun in a jar by running fusion experiments using magnetic fields, therefore to understating how this phenomenon is calculated/measured is indeed very valuable and important to our understanding of the universe and capitalism :) .

In this experiment we will be experimenting with magnetic fields and see how they spread around the source object. This spread is defined as the magnetic field strength and it's unique depending on the source object used.We will experiment with current carrying conductors in different shapes including a ring with radius R, an air solenoid with N turns , radius R and length L (variable), as well as a single Helmholtz coil with N turns and Radius R and finally two Helmholtz coils combined to produce a magnetic field with the principle of superposition separated by a distance x.
The theory behind this experiment is the Biot Savart law which allows us to predict how a magnetic field would form for different geometries, we will compare our experimental results with the theoretical results using the equations provided by source [1], the procedure is discussed in section 3.

3 Procedure

3.1 Equipment [1]

- various connecting cables
- air solenoid with varying length
- axial B-probe
- stand for air solenoid
- teslameter
- holder for conductors
- pair of Helmholtz coils
- assorted conductors
- high current power supply
- stand base with small optical bench

3.2 Methods

In first principles the Method used to measure the magnetic field is by seeing how the reading on the Tesla-meter changes with a varying property of the system, while the way used to preform this measurement differs for different properties (distance,object length,current and Radius) and source objects i.e slightly different orientation of the apparatus for the ring,solenoid, the Helmholtz coil and the Azimuthal and Radial components of the Helmholtz coil, it remains a common theme to use the b-probe to measure the magnetic field as illustrated by figures 1, 2 and 3.

Figure 1: Showing the apparatus for determining B for a ring or a Helmholtz coil [1]

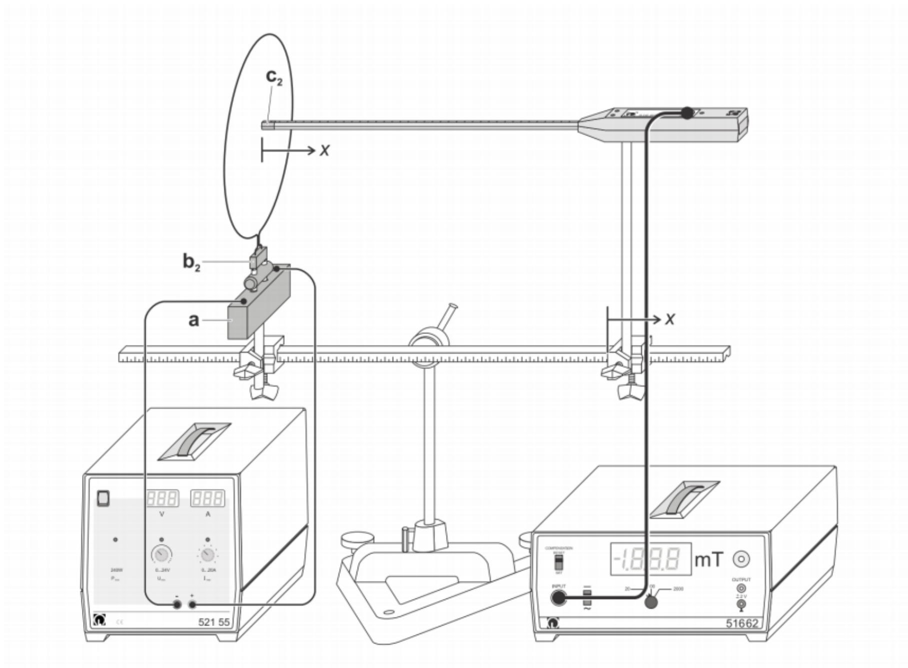


Figure 2: Showing the apparatus for determining B for a straight wire [1]

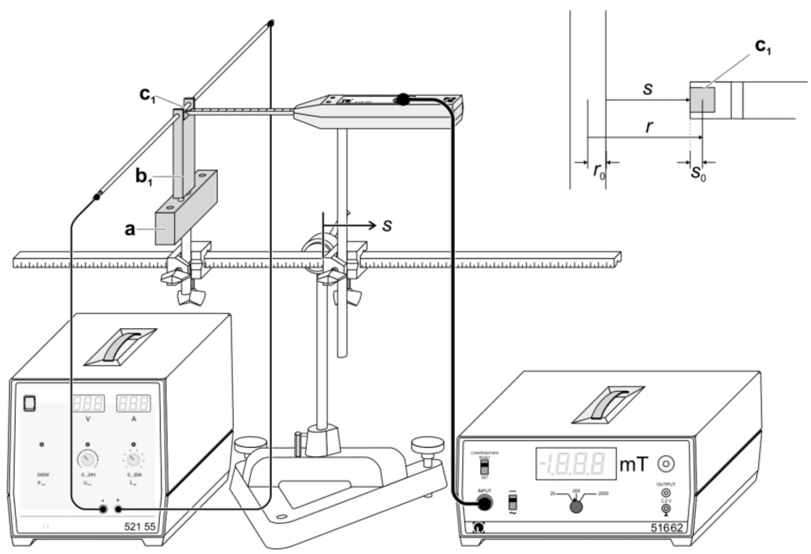
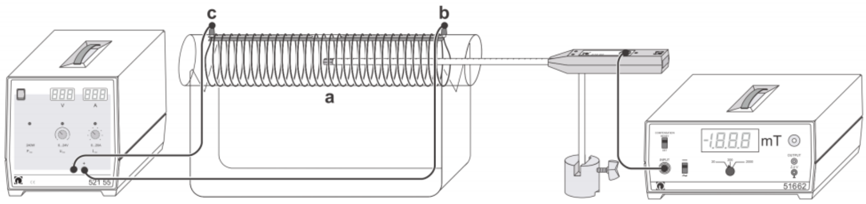


Figure 3: Showing the apparatus for determining B for a solenoid [1]



3.3 Objectives [1]

- 1) Measure the magnetic field due to current carrying conductors, including loops.
- 2) Measure the axial magnetic field of an air solenoid
- 3) Measure the radial and tangential magnetic field of a coil.
- 4) Measure the axial magnetic field of a pair of Helmholtz coils

For objective 1: To measure the magnetic field for a loop/ring we use the set-up in figure 1, the B-probe is place as close to the center of the ring as possible where the distance x in figure 1 is zero, this allows us to measure the varying current and see how that effects the change in the magnetic field at that specific point, on the other hand we will also see how the varying distance x effects the change in the magnetic field strength detected by the B-probe when the current is constant. The current is changed by changing the current dial on the power supply and the distance is changed by moving the coil along the meter track. Before taking any measurements the Tesla-meter is calibrated.This measurement is repeated for 3 different ring radii 40mm,80mm and 120mm and finally it's repeated for a straight wire conductor using apparatus in figure 2.

For objective 2: To measure the axial magnetic field of an air solenoid we used the apparatus in figure 3, the B-probe is inside of the air solenoid at the center of the length (point a), when varying the current the B-probe is fixed at point a and the current is changed from 0-16 A in increments of 2A, on the other hand when varying the distance the current is constant and the distance is changed from a range of -20 cm to 20cm from the center of the solenoid at point a, furthermore when the solenoid length is varied the distance and current is constant and the length is varied from 10cm - 40cm in increments of 5cm. When constant the distance is kept at 0cm from the center point a, the current at 16cm and solenoid length is at 15cm.

For objective 3: To measure the radial and tangential/Azimuthal magnetic field of a coil the setup in figure 1 was used, however instead of the ring a Helmholtz coil is used and the orientation of the B-probe is changed to measure both the radial and tangential components as the Helmholtz coil is rotated 360 degrees. The magnetic field is measured every 30 degrees.

For objective 4: To measure the axial magnetic field of a pair of Helmholtz coils two Helmholtz coils were placed along the meter track, looking at figure 1, instead of one ring two Helmholtz coils are placed along the track and the magnetic field is investigated for different separation lengths between the two coils at suitable distance intervals i.e we measure the magnetic field from a suitable length $\pm$ X where the center point is the middle of the separation between the two coils, this measurement is repeated for 3 different separations where one is the optimum and the other two are below and above the optimum.

For all objectives result plottings were made and where appropriate the predicted theoretical curve was also plotted along side the experimental data, this is included in section 4 and further discussed in section 5.

4 Results

4.1 Magnetic field due to current carrying conductors

Figure 4: BvI and theoretical

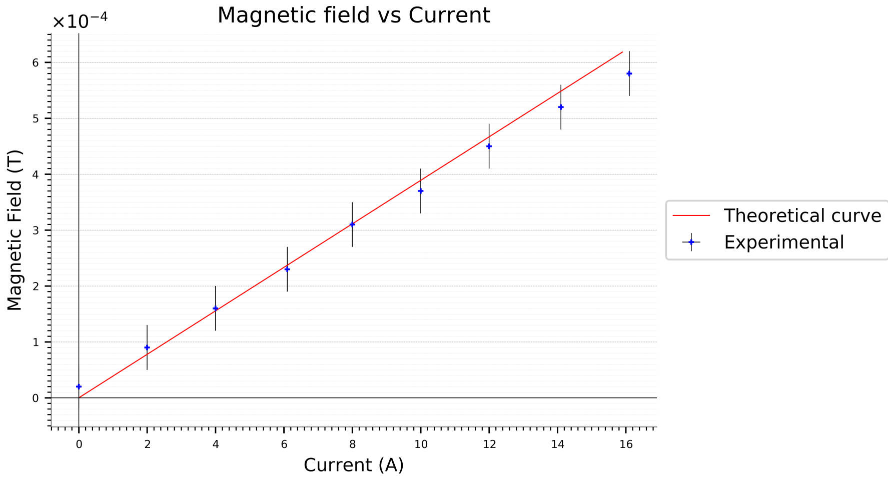


Figure 5: BvX and theoretical (Constant current) 40mm conductor

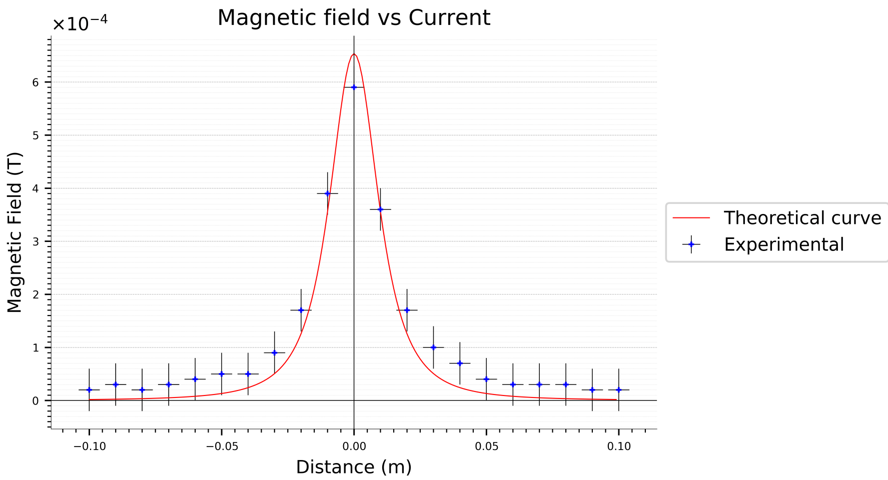


Figure 6: BvX and theoretical (Constant current) 80mm conductor

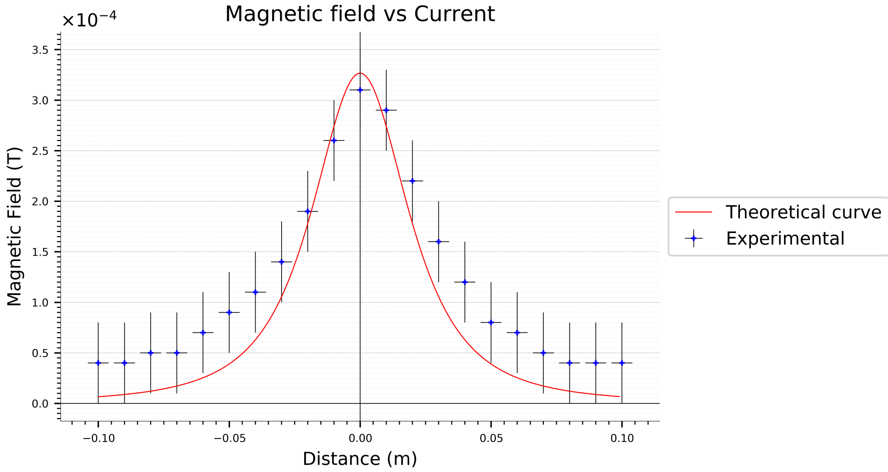


Figure 7: BvX and theoretical (Constant current) 120mm conductor

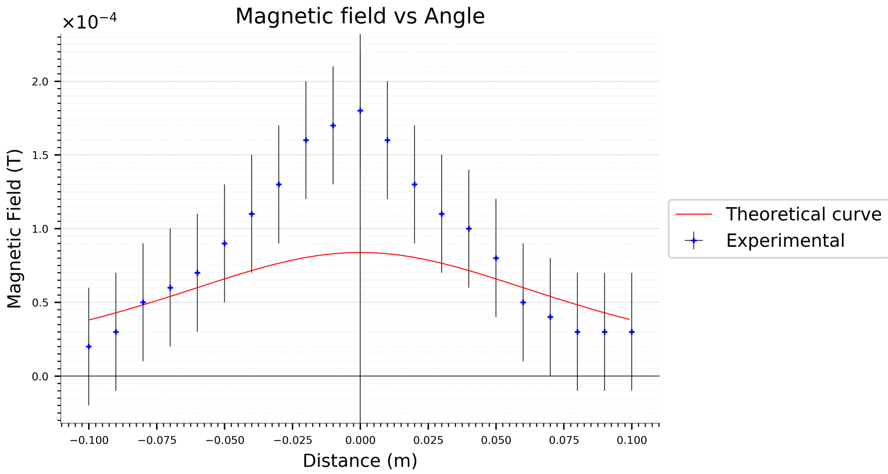


Figure 8: BvX and theoretical (Constant current) 120mm conductor results vs 55mm theoretical results (potential error fit)

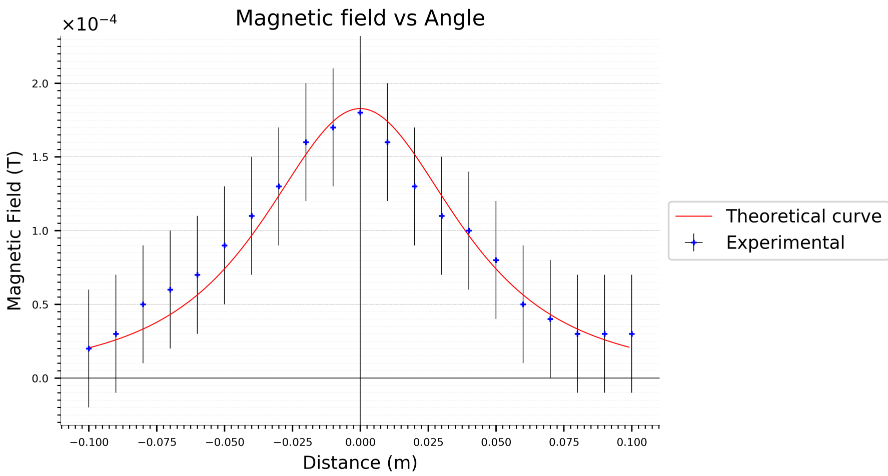
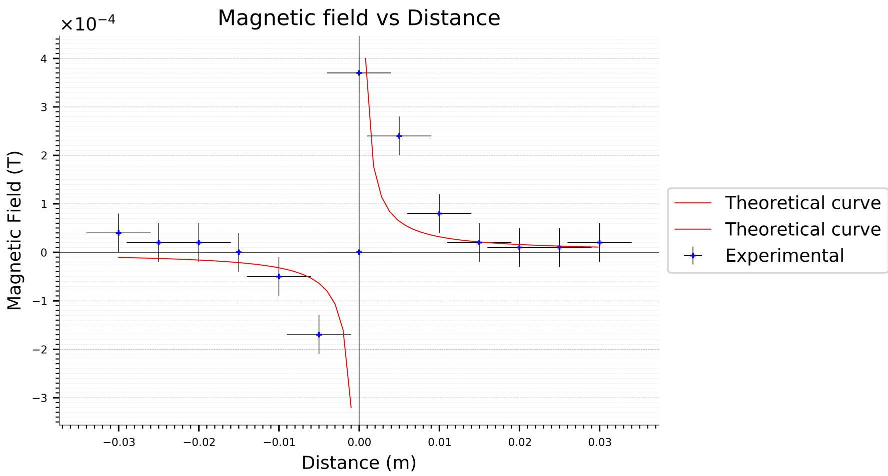


Figure 9: BvX and theoretical (Constant current) Long wire



4.2 Axial magnetic field of an air solenoid

Table 1: Error analysis for B v Current

Current (A)	Magnetic field (mT)	uncertainty (=-T)
1.60E+01	4.35E+00	1.09E-04
1.40E+01	3.84E+00	9.60E-05
1.20E+01	3.22E+00	8.31E-05
1.00E+01	2.70E+00	7.03E-05
8.00E+00	2.17E+00	5.77E-05
6.00E+00	1.58E+00	4.57E-05
4.00E+00	1.12E+00	3.46E-05
2.00E+00	5.10E-01	2.57E-05
0.00E+00	2.00E-02	2.19E-05

Figure 10: BvI and theoretical

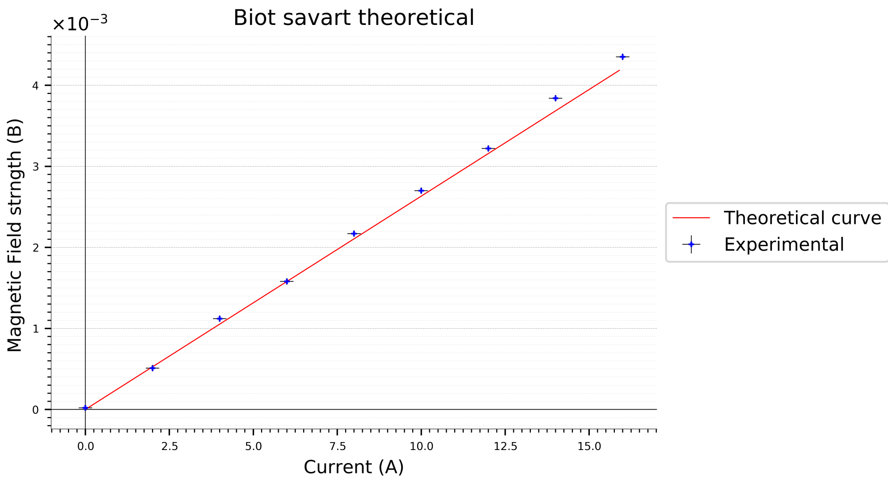


Table 2: Error analysis for B v Solenoid length (l)

Lair	Magnetic Field (mT)	uncertainty
1.00E+01	4.97E+00	3.79E-07
1.50E+01	3.98E+00	2.52E-07
2.00E+01	2.93E+00	1.89E-07
2.50E+01	2.52E+00	1.51E-07
3.00E+01	2.09E+00	1.26E-07
3.50E+01	1.98E+00	1.08E-07
4.00E+01	1.97E+00	9.43E-08

Figure 11: BvL and theoretical

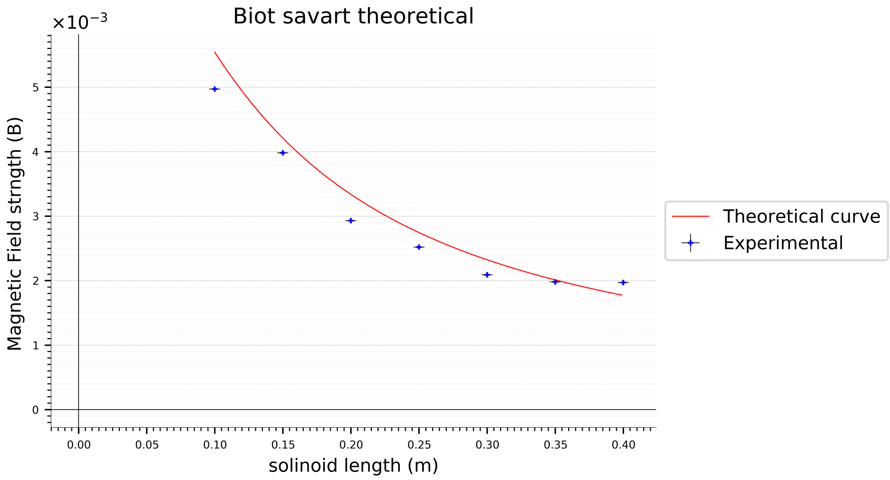
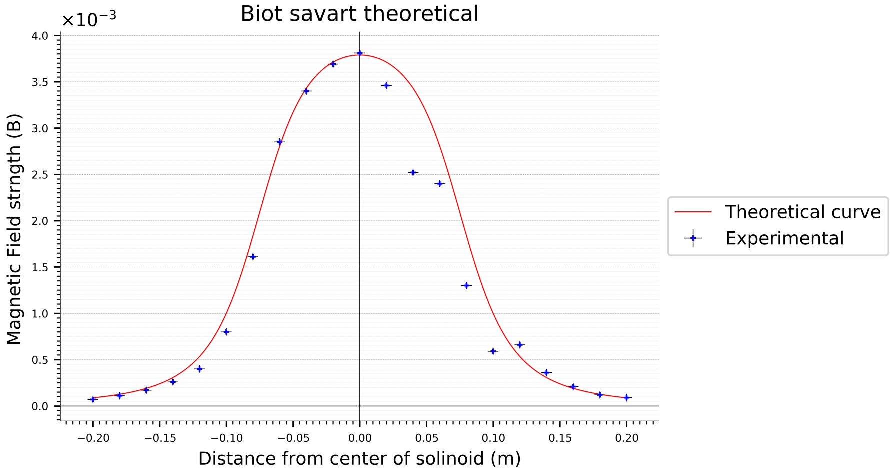


Table 3: Error analysis for B v Distance

Distance (cm)	B (mT)	Uncertainty in B (+- T)
2.00E+01	9.00E-02	4.58E-06
1.80E+01	1.20E-01	7.54E-06
1.60E+01	2.10E-01	1.33E-05
1.40E+01	3.60E-01	2.54E-05
1.20E+01	6.60E-01	5.20E-05
1.00E+01	5.90E-01	1.05E-04
8.00E+00	1.30E+00	1.57E-04
6.00E+00	2.40E+00	1.19E-04
4.00E+00	2.52E+00	8.40E-05
2.00E+00	3.46E+00	1.01E-04
0.00E+00	3.81E+00	1.09E-04
-2.00E+00	3.69E+00	1.01E-04
-4.00E+00	3.40E+00	8.40E-05
-6.00E+00	2.85E+00	1.19E-04
-8.00E+00	1.61E+00	1.57E-04
-1.00E+01	8.00E-01	1.05E-04
-1.20E+01	4.00E-01	5.20E-05
-1.40E+01	2.60E-01	2.54E-05
-1.60E+01	1.70E-01	1.33E-05
-1.80E+01	1.10E-01	7.54E-06
-2.00E+01	7.00E-02	4.58E-06

Figure 12: BvX and theoretical



4.3 Radial and tangential magnetic field of a coil

Figure 13: Radial direction Bvθ and theoretical (constant current)

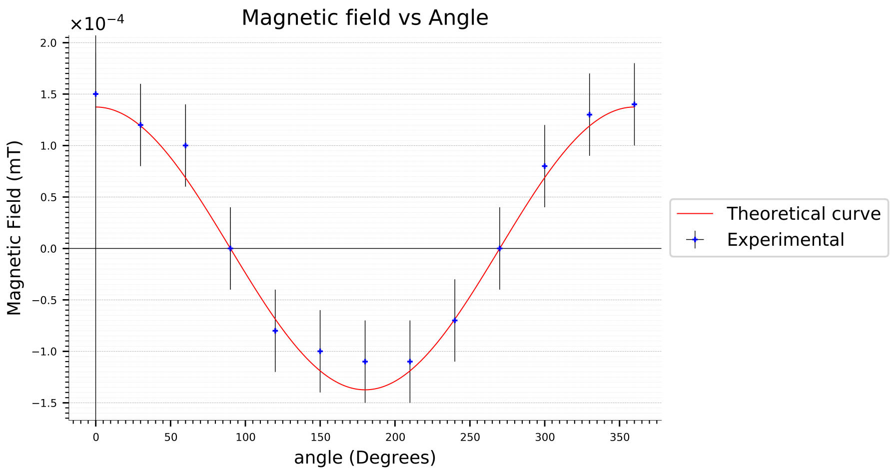
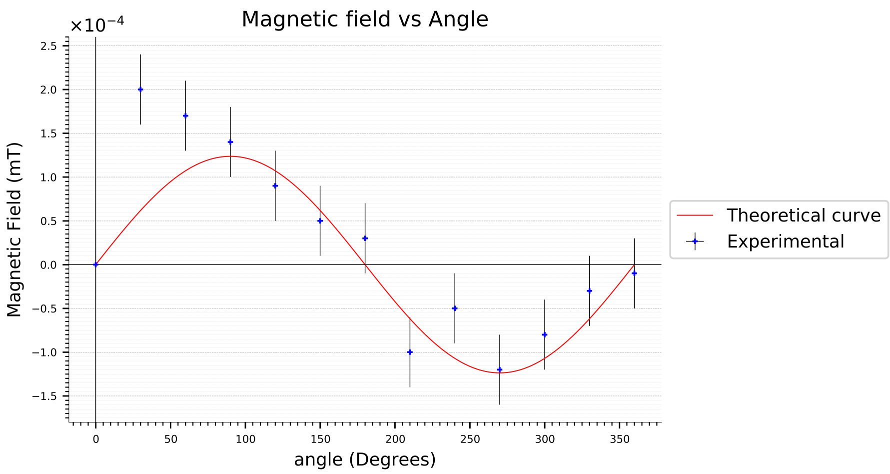


Figure 14: Azimuthal direction Bvθ and theoretical (constant current)



4.4 Axial magnetic field of Helmholtz coils

Figure 15: Magnetic field of a single helmholtz coil vs distance

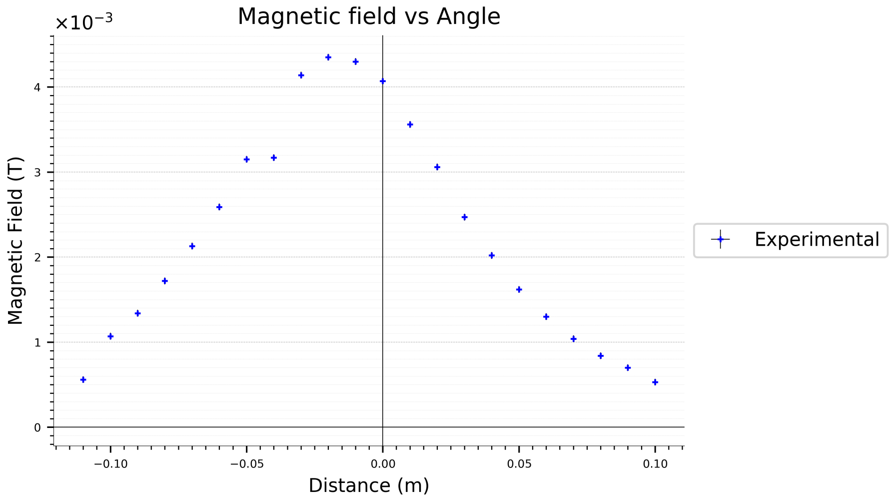


Figure 16: 5cm separation low

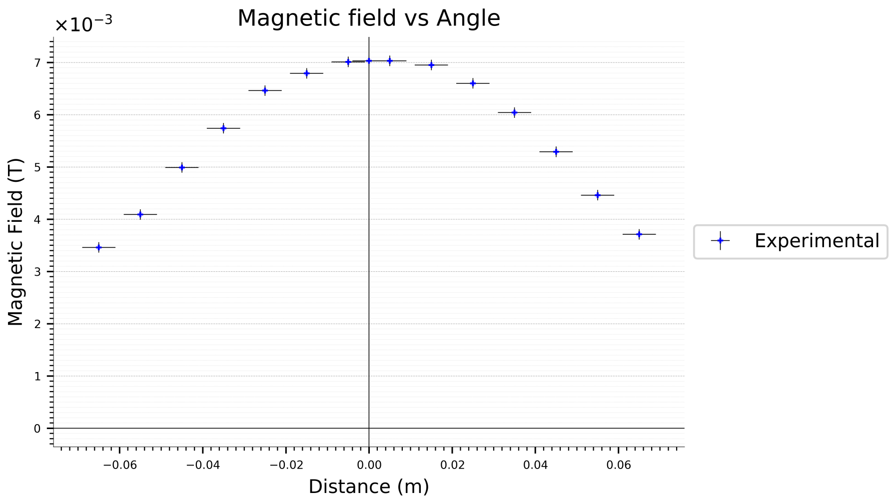


Figure 17: 10cm separation high

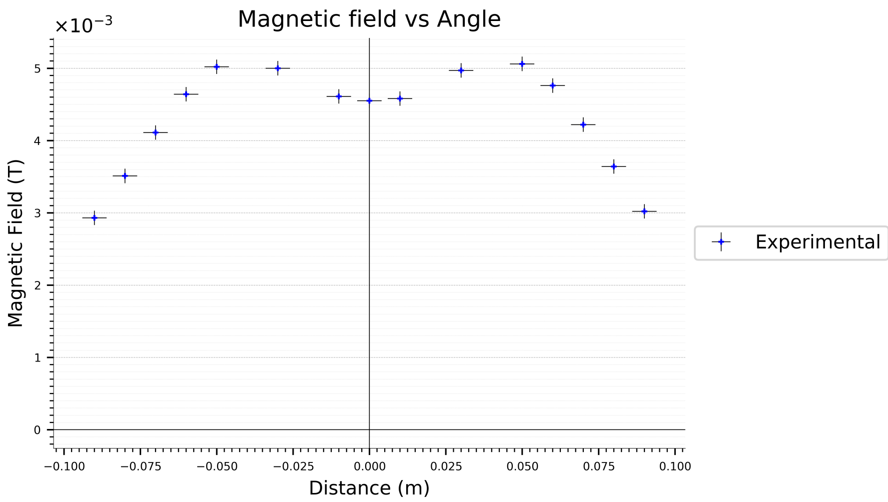
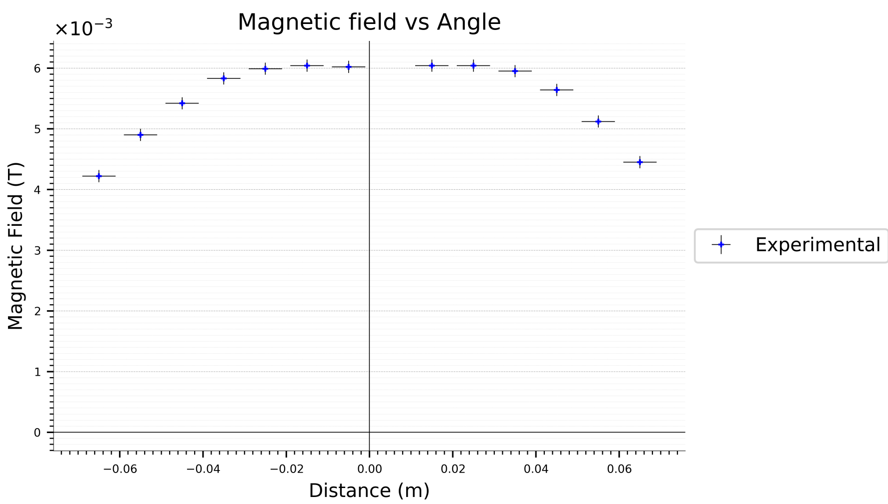


Figure 18: 7cm separation optimal



5 Discussion

For section 4.1 figure 4 the current is varied from 0-16A in increments of 2 A, compared with the theoretical curve obtained by the Biot Savart law using equation 1, the figure shows that the experimental results accurately matches the theoretical curve achieved by the Biot savart law equation. Furthermore for figures 5,6 and 7, we can see how the magnetic field strength changes with the distance on both sides of the source object and by looking the different figures we can see how the radius of the ring effects how the magnetic field strength curve with respect to distance changes. The theoretical curves on these graphs are obtained by changing the radius for each and iterating the value of x from -10cm -> 10cm using equation 1, while having all other values constant. We can see that as the radius increases from 20mm-80mm-120mm the peak of the curve decreases and all curve flattens out, suggesting that at smaller radii the magnetic field has a greater intensity at all points and as the radius increases the magnetic field strength or the 'field lines' unravel and thus the magnetic field is weaker.

$$B = \frac{\mu_0 I 2\pi R^2}{4\pi(R^2+x^2)^{3/2}} \tag{1}$$

Furthermore for figure 9 we can see the magnetic field distribution from a current carrying wire at a distance x from the center of the wire, measurements were taken for a distance $\pm 3cm$ at 0.5cm intervals and compared with the theoretical curve, we can see that as the distance increases the magnetic field strength drops exponentially when moving away (perpendicularly) from the wire, this compared to the ring shows two completely different behaviours of the magnetic field due to the geometry of the source object, when moving in a perpendicular direction around a ring the magnetic field strength wasn't decreasing as quickly compared to the wire, suggesting the the geometry of the object results in a different shape of the magnetic field.

For figure 10 we can see how the magnetic field strength changes as the current inside the air solenoid changes when the distance from the center of the solenoid is constant. We can see that the relationship is linear which is similar to the result obtained in figure 4, suggesting that for different geometries the incremental change in current will result in a linear increase in the magnetic field strength regardless of the geometry used. Furthermore for the solenoid in figure 11 the iterated variable is the solenoid length it-self, we observe that as the solenoid length increases the magnetic field strength at the center point of the solenoid length decreases in a curve that appears to be a parabola, again suggesting the 'unraveling' of the magnetic field lines and therefore the strength of the field as discussed before for figures 5,6 and 7.

On the other hand when the distance of the probe is iterated, in figure 12 we can see a bell shaped distribution graph where the peak is at the center of the solenoid length and the magnetic field strength decreases as the distance changes, this describes another way in which magnetic field distribution is formed due to the geometry of the current carrying source, compared to figures figures 5,6 and 7, we can see that this distribution (Theoretical) graph which was plotted using the equation in figure 19 is more uniformly distributed around the center point of the solenoid, suggesting that as the source object spreads further into the perpendicular direction from the direction of travel of the charge i.e as a circle becomes a tube the magnetic field strength at the points in the perpendicular directions becomes stronger due to the principle of superposition where the magnetic field strength increases at a certain point due to all other magnetic field line generated at other points and this magnetic field strength is greatest at the center point where the superposition is greatest.

Figure 19: Biot savart law for the solenoid [1]

$$B = \mu_0 \, I \, \frac{N}{2\,L} \left(\frac{x + \frac{L}{2}}{\sqrt{\left(x + \frac{L}{2}\right)^2 + R^2}} - \frac{x - \frac{L}{2}}{\sqrt{\left(x - \frac{L}{2}\right)^2 + R^2}} \right),$$

For figures 13 and 14 we can see how the magnetic field strength produces by the Helmholtz coil is effected in both the radial and azimuthal directions when the coil is rotated 360 degrees, the theoretical curves were plotted using the equation in figure 20 and they show a cosine curve for the radial and sine curve for the azimuthal suggesting that that phase difference between the two orientations is 180 degrees suggesting that the two measured magnetic fields are perpendicular to each other, on the other hand the behaviour observed again is different but similar to the current carrying rings and the solenoid, since we now have many rings on top of each other and that is repeated along the perpendicular direction to the rings i.e forming a hollow solid cylinder of charge carrying rings we obtain a much stronger magnetic field in both directions radial and azimuthal both of which are present in previous orientation but due to the principle of superposition they are now much stronger. We can see that as the Helmholtz coil is rotated the magnetic field is again strongest when directly in front of the opening of the coil and lowest at the edge of the coil for the radial directions and exactly the opposite for the azimuthal direction therefore demonstrating how the magnetic field is shaped around the Helmholtz coil i.e the radial direction is a tube of decreasing magnetic field strength with distance and the azimuthal direction is like two half rings of decreasing magnetic strength from the sides of the coil where the maximum is at the ± 90 degrees from the open center of the coil. Furthermore the decrease in the magnetic field strength as a rotation occurs again illustrated the idea of the decrease in the magnetic field strength as a variables change be it distance of angle.

Figure 20: Biot savart law for the Helmholtz coil in the radial and azimuthal directions [1]

$$B = \mu_0 \, I \, \frac{N}{2\,L} \left(\frac{x + \frac{L}{2}}{\sqrt{\left(x + \frac{L}{2}\right)^2 + R^2}} - \frac{x - \frac{L}{2}}{\sqrt{\left(x - \frac{L}{2}\right)^2 + R^2}} \right),$$

Furthermore for figures 15,16,17,18.Figure 15 shows the magnetic field strength as the distance from one Helmholtz coil increases from ± 10cm from the center point which is located at the center of the Helmholtz coil, again showing the unraveling of the magnetic field lines.
On the other hand figures 16,17,18, show how the magnetic field of two Helmholtz coils is formed depending on the distance between the coils, for figure 16 the distance between the coils is very small and due to the principle of superposition we see that this particular curve has a peak compared to figure 17 which shows a dip, suggesting that constructive and destructive interference are operating on respectively on the coils at the different distances, on the other hand when we reach a point of even constructive interference/the optimal distance where the magnetic field inside of the space between the two coils is the same at every point therefore explaining the flat line in the middle of figure 18.

Errors in the experiment has occurred that may describe of the inconsistencies in figures 7,14 and 15, this is further discussed in section 6.

6 Error analysis

To analyze some the errors a closer look at figures 7,14 and 15 we should have.

For figure 7 we see that the theoretical curve is much flatter than the experimental results, the potential sources of this error is the false use of a ring labeled as 120mm when it's not, as illustrated by a plotting made where the theoretical radius is made to be 55mm we see that the theoretical curve accurately fits the experimental results in figure 8, on the other hand this error may have also resulted if unknowingly the B-probe was not well calibrated or if it was mis-calibrated by accident resulting in a completely failed curve.
Furthermore for figure 14, we see that the two measurements taken are anomalies that do not fit the theoretical curve this may have resulted because of a glitch in the B-probe or because the initial current was high due to human error, however it's most likely that a simple misreading of the Tesla-meter happened which also classifies as human error, this is also most likely the case for the anomaly in figure 15.

7 Conclusion

To conclude this experiment shows us how the magnetic field is formed in and around different current carrying sources from rings, a straight wire and to a solenoid then finally Helmholtz coils.We clearly explored the idea that the magnetic field strength is heavily dependant on many features and the relationship between the magnetic field strength and that feature e.g a change in current is linear with a change in the magnetic field while an increase in distance results in the decrease of the magnetic field, as well as looking at the concept of super position and the unraveling' of the magnetic field strength with two Helmholtz coils and varying (angle,distance and length of an object) respectively.

8 Questions

Question one: after analyzing the magnetic field coming from every direction within the lab we discovered that the greatest difference in the magnetic field strength occur at an angle of 240 degrees $\pm 30 degrees$ relative to the entrance door to where the milikan's experiment takes place, later this direction with the uncertainty lies within the north direction i.e it lies on the direction torwards wich the earth magnetic field is present.

Question two: If the direction of current is reversed in one the coils the magnetic field inside will experience destructive interference therefore the magnetic field inside will become zeros however if both are reversed the magnetic field will have the same magnitude but in the negative direction

9 Python Code

Python was used to generate all the graphs as well as calculating the error propagation listed on this report, the python code was generated as a collaboration with the named lab partner and cannot be included in this report due to length issues however it saved in : var/user/mo379/physics/python/grapher.py and var/user/mo379/physics/python/uncertainty_calculator.py

References

[1] UKC SPS. “Biot-Savart-Law labscript”. In: (2018). URL: file://bodiam/AYT/mo379/My%5C%20Files/Downloads/Biot_Savart_s_Law_Sept%5C%202018%5C%20(1).pdf.

[2] wiki. “Magnetism”. In: (?). URL: https://en.wikipedia.org/wiki/Magnetism.